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RADIANT HEAT TRANSFER IN A FURNACE WITH TWO VOLUME ZONES

S. P. Detkov

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A modification of the model of radiant heat transfer in a furnace for arbitrary transmission of the furnace core is proposed.

1. Introduction

In the mathematical model of [1], a furnace is represented by a cylindrical channel with division along the axis into zones with isothermal volumes in each section. Of course, the model gives significantly overestimated values of the heat transfer or underestimated values of the temperature of the exhaust gases, other conditions being equal. In [2], the model was significantly improved. The volumes in the radial direction are divided into two coaxial layers: the core and a conservative part; the conservative layer (CL) is nonisothermal. Essentially, the core is also nonisothermal, but is characterized by the mean (over the cross section) pyrometric temperature. Some deficiencies of the model may be noted: a) the core is assumed to be optically dense and is replaced by a nontransparent surface with equivalent radiational properties; the model corresponds to a large furnace; b) the spectrum in extreme representations (grey and antigrey) only changes in the CL.

The present model is modified on a new basis. The furnace core may have any transmission characteristics; therefore the model in fact has two volume zones in each cross section of the channel.

The principal underlying this new zonal-calculation approach was outlined in [3, 4]. The volume of the medium is replaced by a surface with equivalent radiative properties. This surface transmits some of the incident flux. Since the volume has a real temperature field, the surface has different local values of the intrinsic-radiation density q_c and other quantities. Therefore, it is divided into zones with mean internal characteristics. In the present work, in contrast to [3-5], the volume is divided into two zones, and the

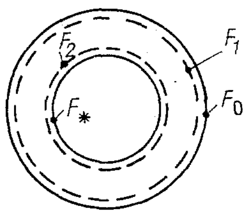

Fig. 1. System of four surfaces: three (F_1, F_2, F_*) represent two volume zones: the core and the conservative layer.

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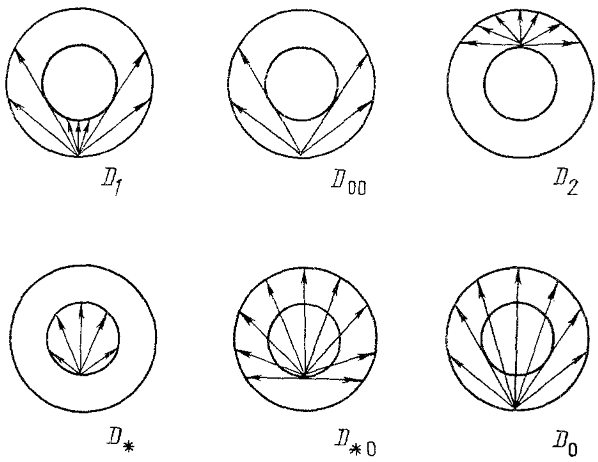

Fig. 2. Fans of arrows denoting the volumes for which the transmission capacities D_1, D_{00}, D_2, D_*, D_{0*}, D_0 are determined.

method takes on additional possibilities. In the axisymmetric model with three zones (two volume zones and the shell), four surface zones and correspondingly a system of four zonal equations are obtained.

2. System of Bodies

The shell F_0 with absorptive capacity A_0, the external surface $F_1 = F_0$ of the annular layer with transmissive, absorptive, and reflective capacities D_1, a_1, r_1, the internal surface F_2 with components D_2, a_2, r_2, and the external surface of the core $F_* = F_2$ with constants D_*, a_*, r_* are shown in Fig. 1. The quantities r_i appear in acts of energy scattering. The ratio $F_*F_0 = \varphi$ is the angular coefficient at surface F_* in the absence of a medium. In Fig. 2, fans of arrows denote the volumes for which the integral transmissive capacities D_1, D_{00}, D_2, D_*, D_{*0}, D_0 and other optical characteristics are determined. It is simple to prove that

$$D_{*0} = D_2 D_*;\ D_0 = D_1 - \varphi D_2 (1 - D_{*0});$$
$$(1 - \varphi) D_{00} = D_0 - \varphi D_2 D_* = D_1 - \varphi D_2. \qquad (1)$$

The following relations are employed: $a_1 + r_1 + D_1 = 1$; $a_2 + r_2 + D_2 = 1$; $a_* + r_* + D_* = 1$; $A_0 + R_0 = 1$. The emissivities and absorptive capacities are assumed to be equal: $\varepsilon_1 = a_1$; $\varepsilon_2 = a_2$; $\varepsilon_* = a_*$; $\varepsilon_0 = A_0$.

3. Zonal Equations

The densities of effective and intrinsic fluxes for the surface F_i are related by the matrix equation

$$(I - R\psi)\, q_e = q_c, \qquad (2)$$

where R is the diagonal matrix of reflective capacities. In writing the matrices in the order F_0, F_1, F_2, F_*, the matrix $I - R\psi$ takes the form

$$\begin{pmatrix} 1 - R_0 D_0 & -R_0 & -R_0 \varphi D_{*0} & -R_0 \varphi D_2 \\ -r_1 & 1 & 0 & 0 \\ -r_2 D_{*0} & 0 & 1 - r_2 D_* & -r_2 \\ -r_* D_2 & 0 & -r_* & 1 \end{pmatrix}$$

Multiplication of the matrices in Eq. (2) gives the system of zonal equations

$$(1 - R_0 D_0)\, q_{e0} = q_{c0} + R_0 q_{e1} + R_0 \varphi D_{*0} q_{e2} + R_0 \varphi D_2 q_{e*}, \qquad (3)$$

$$q_{e1} = q_{c1} + r_1 q_{e0}, \qquad (4)$$

$$(1 - r_2 D_*)\, q_{e2} = q_{c2} + r_2 D_{*0} q_{e0} + r_2 q_{e*}, \qquad (5)$$

$$q_{e*} = q_{c*} + r_* D_2 q_0 + r_* q_{e2}. \tag{6}$$

In the case of a black shell and with no energy scattering $R_0 = r_i = 0$, the equations degenerate to trivial equalities $q_{ei} = q_{ci}$. Hence it follows that they are inadequate for the solution of heat-transfer problems.

In calculating q_{ei}, the error may be large because of the indeterminancy in r_i. First, the reflective capacities of the zones are small; the value r = 0.05 in the illustrative calculation below is close to the maximum for commercial furnaces, even large ones. Second, they are usually estimated with low accuracy. For such cases, the system must be transformed.

According to the classification of radiant fluxes for the shells (surface F_0)

$$A_0 q_{e0} = R_0 q_{r0} + q_{c0}. \tag{7}$$

For the surface bounding the volume zone, for example, for F_* with transmissive capacity D_*

$$a_* q_{e*} = r_* q_{r*} + (1 - D_*) q_{c*}. \tag{8}$$

Equating the right-hand sides of Eqs. (6) and (8) with division of Eq. (8) by a_* gives

$$q_{c*} - q_* = a_* (q_{e2} + D_2 q_{e0}), \quad q_* = - q_{r*}. \tag{9}$$

For surface F_2

$$a_2 q_{e2} = r_2 q_{r2} + (1 - D_2) q_{c2};$$

substitution of q_{e_2} into Eq. (5) gives

$$(1 - r_2 D_*) q_{r2} + [1 - D_* (1 - D_2)] q_{c2} = a_2 (q_{e*} + D_{*0} q_{e0}). \tag{10}$$

By the same method, the following equation is obtained for surface F_1 instead of Eq. (4)

$$q_{r1} + q_{c1} = a_1 q_{e0}. \tag{11}$$

Now r_1 does not play a large role if it is multiplied by quantities which are known or obtained from additional equations. Analogous transformations are performed with Eq. (3) as $R_0 \to 0$. After substitution of q_{e_0} from Eq. (7), it is found that

$$(1 - R_0 D_0) q_{r0} + (1 - D_0) q_{c0} = A_0 (q_{e1} + \varphi D_{*0} q_{e2} + \varphi D_2 q_{e*}). \tag{12}$$

The systems in Eqs. (3)-(6) and Eqs. (9)-(12) admit of other equivalent transformations on the basis of the classification of the radiant fluxes as a function of the formulation of the problem.

Radiation sources may act in both zones. However, the condition that the external layer is conservative is introduced below, as in [2]. This gives the additional equations

$$q_* = q_{r0}/\varphi, \quad q_{r2} = - q_{r1}/\varphi, \text{ where } q_* = - q_{r*}. \tag{13}$$

Other phenomenological equations are also useful. For example, the equations

$$q_{r0} = q_{e1} + q_{I2} \varphi D_2 - q_{e0} [1 - (1 - \varphi) D_{00}], \quad q_* = q_{I2} - q_{e2} - D_2 q_{e0}$$

lead to the following relation after elimination of q_{I_2} and the substitution of D_{00} from Eq. (1) and q_{r_0} from Eq. (13)

$$\varphi (1 - D_2) q_* + [1 - D_1 + \varphi D_2 (1 - D_2)] q_{e0} = q_{e1} + \varphi D_2 q_{e2}. \tag{14}$$

With specified conditions at the surface F_0 (θ_0, A_0, q_{r_0}) and φ, the left-hand side of Eq. (14) is constant with any choice of q_{e1} and q_{e2}. It is evident that the underestimation in the calculation of q_{e_1} below leads to overestimation of q_{e_2} and, according to Eq. (9), overestimation of q_{c*} and the core temperature θ_*. Usually, the reflective capacity of the CL is small: $r_1 \simeq r_2 \simeq 0$; $q_{e2} \simeq q_{c2}$ and $q_{e2} \simeq q_{c1}$. If $q_{e1} = \varphi q_{e2}$ is assumed as a rough approximation, it follows from Eq. (14) that

$$q_{e1} = \{\varphi(1-D_2)q_* + [1 - D_1 + \varphi D_2(1-D_2)]\,q_{e0}\}/(1+D_2). \tag{15}$$

In this case, the balance equations are adequate for the solution of the problem. However, the estimate in Eq. (15) must be refined. The system of balance equations is satisfied by a set of q_{e_1} and only one of them does not conflict with the internal heat transfer of the volume. Simple estimates of q_{c_1} are given below. Note that q_{e_1} may be found directly from Eq. (4).

4. Estimate of the Flux q_{c1}

The estimate of the intrinsic radiation of the annular zone at the shell is given first of all in the plane-layer approximation. It was shown in [6] that the semiinfinite conservative medium may be replaced by a boundary layer with optical thickness $\tau_0 \gtrsim 0.5$ and an equivalent surface established at this depth. If there is no energy scattering, then

$$q_{c1} = 2\int_0^{\tau_0} \theta_1^4 D\,d\tau,$$

where d = $\exp(-1.8\tau)$ on the basis of the effective depth. According to [6], with a cold black wall

$$\theta_1^4 = 0.75 q_{r0}[\tau + f(\tau)],\ f(\tau) = 0.7104 - 0.1331e^{-3.7\tau}.$$

Substitution of $\theta_1{}^4$ and replacement of τ_0 according to the relation $1.8\tau_0 = \ln(1/D_2)$ gives the result

$$q_{c1}^0 = 0.83 q_{r0}\{0.71(1-D_2) + 0.53[1 - (1 + \ln\langle 1/D_2\rangle)D_2] - 0.04(1-D_2^3)\}.$$

The coefficients are slightly corrected, so that

$$q_{c1}^0 \to q_{r0} \text{ as } D_2 \to 0.$$

In a heated grey shell with thermodynamic equilibrium, the temperature is established from the relation $\theta_{10}{}^4 = q_{e_0}$. Therefore, $\theta_{10}{}^4$ becomes the term with θ^4 in the integrand. Repeated integraton of $q_{c_1}{}^0$ corresponds to the addition of the term $q_{e_0}(1 - D_2)$, in which the coefficient 2/1.8 is omitted. It remains to take account of the energy scattering.

In this case, with thermodynamic equilibrium, the term $q_{e_0}(1 - D_2)$ determines q_{e_1}. It is found that $q_{c_1} = q_{e_0}(1 - D_2 - r_1)$. In the general example, as in the given particular case, the substitution $D_2 \to D_2 + r_1$ is made. Finally

$$q_{c1} = q_{e0}(1 - D_2 - r_1) + 0.83 q_{r0}\Big\{0.71(1 - D_2 - r_1) + 0.53\left[1 - \left(1 + \ln\frac{1}{D_2 + r_1}\right)(D_2 + r_1)\right] - 0.04[1 - (D_2 + r_1)^3]\Big\}. \tag{16}$$

If the energy is scattered without absorption, then $D_2 + r_1 = 1$ and $q_{c_1} = 0$.

5. Influence of the Curvature of the Layer on q_{c1}

Consider the simplest example of an optically dense core with its surface at the same temperature θ_*. The surface F_0 is black and cold, and there is no energy scattering in the medium ($R_0 = r_i = 0$). According to [7]

$$q_{r0} = [\varphi/(1 + \xi_{*0})]\,\theta_*^4, \tag{17}$$

where the thermal resistance of the CL is described by the formula

$$\xi_{*0} = 0.75\tau_0 \ln(1/\varphi)/[(1/\varphi) - 1],\ \tau_0 = k(\rho_0 - \rho)$$

and k is the attenuation coefficient of the grey medium.

With specified q_{r_0} and τ_0, increase in curvature of the layer (decrease in φ) is accompanied by increase in θ_* and hence q_{c*}. According to Eq. (9), q_{c_2} also increases. In the given particular case, $q_{ei} = q_{ci}$, $q_{e_0} = 0$. According to Eq. (14)

$$a_2 q_{r0} = q_{c1} + \varphi D_2 q_{c2}. \tag{18}$$

Using Eq. (9)

$$\varphi q_{c*} - q_{r0} = \varphi a_* q_{c2}$$

and eliminating φq_{c2} it follows that

$$(a_2 a_* + D_2) q_{r0} = a_* q_{c1} + a_* D_2 \varphi \theta_*^4 .$$

Assuming $a_* = 1$ and eliminating $\varphi\theta_*^4$ from Eq. (17), in which $a_* = 1$ also, it is found that

$$q_{c1}^0 = [1 - D_2(1 + \xi_{*0})] q_{r0}. \tag{19}$$

For a plane layer, $D_2 \approx \exp(-1.8\tau_0)$; hence $\tau_0 = 0.2838$ when $D_2 = 0.6$. Taking into account that $\varphi = 1$, $q_{c1} = 0.2723 q_{r0}$. Curvature of the layer with $\varphi = 0.6$ with the same optical thickness increases q_{c1} on account of the reduction in ξ_{*0}, $q_{c1} = 0.3021 q_{r0}$; i.e., the increase is 11%. In fact, D_2 increases; therefore, the increase in q_{c1} is less. According to the estimate in Eq. (16), $q_{c1}{}^0 = 0.2508 q_{r_0}$ when $r_1 = q_{e0} = 0$. One more estimate is associated with the assumption $q_{c1} \lesssim \varphi q_{c2}$. This assumption results from passing in the limit to a plane layer, when $\varphi = 1$. At large D_2, the right- and left-hand sides are little different, and the sign of the inequality may change with decrease in φ. As a result of formulating Eq. (18), it follows that

$$q_{c1}^0 = [(1 - D_2)/(1 + D_2)] q_{r0} = 0.25 q_{r0}.$$

6. Problem

Consider an example of the formulation and solution of the problem of radiant heat transfer. The optical characteristics of the volume zones depend on their dimensions. If the dimensions are to be determined, as in the model of [2], the values of the constants are specified in the first-cycle approximation, and refined in the subsequent cycles of the calculation. On the basis of practical experience of furnace operation, the density of the resulting flux at the heat-receiving surface q_{r0} must be specified. From estimates of the contamination, the temperature θ_0 and absorptive capacity A_0 are determined. Having calculated $q_{c0} = A_0\theta_0{}^4$, q_{e0} is found from Eq. (7). It follows from Eq. (13) that $q_* = q_{r0}/\varphi$, $q_{r*} = -q_*$. After estimating q_{c1} from Eq. (16), it is adjusted upward as in the example in Sec. 5. Using Eq. (4), q_{e1} is found, while q_{e2} is determined from Eq. (14). Equation (9) gives q_{c*} and then θ_*. The temperature field in the core may be inhomogeneous; the mean value of θ_* may be called the effective core temperature; it is important that it is obtained in the course of solving the problem. The remainder of the equations are used as controls. It is simple to establish that $q_* \leq q_{c*} \leq a_*$.

7. Numerical Calculation

In the calculations, the conditions are simplified; reflection and scattering of the fluxes are omitted: $R_0 = r_1 = r_2 = r_* = 0$. The general Eq. (2) and the particular Eqs. (3)-(6) degenerate to the trivial equality $q_{ei} = q_{ci}$. The information is contained in the system in Eqs. (9)-(12). The following values are specified: $\theta_0 = 0.2$; $q_{r0} = 0.12$; $\varphi = F_*/F_0 = 0.6$; $a_1 = a_2 = a_* = 0.4$; $D_1 = D_2 = D_* = 0.6$. It follows from Eq. (1) that $D_{*0} = 0.36$; $D_0 = 0.37$. Then $q_* = 0.12/0.6 = 0.2$; $q_{c0} = \theta_0{}^4 = 0.0016$. According to Eq. (16), q_{c1} is 0.03074. The influence of the curvature is disregarded. The results obtained are as follows

$$q_{c0} = 0.0016;\ q_{c1} = 0.03074;\ q_{c2} = 0.05036;\ q_{c*} = 0.2205;\ q_{r0} = 0.12;$$
$$q_{r1} = -0.03010;\ q_{r2} = 0.05017;\ q_{r*} = -0.2;\ \theta_*^4 = 0.5513;\ \theta_* = 0.8617.$$

The mean CL temperature is estimated from $\bar{q}_c = (q_{c1} + q_{c2})/2 = 0.4055$; $\theta^4 = \bar{q}_c/a_1 = 0.1014$; $\bar{\theta} = 0.5643$. In accordance with [2], the temperature field in the furnace is determined by the thermal-radiation mechanism, regardless of convection. The values of θ_* and $\bar{\theta}$ give the exhaust-gas temperature.

8. Convective Part of the Model

In the convective part, the model in [2] is retained. The density of the convective flux at the surface F_0, q_C, is added to the density of the radiant flux $q_{R_0} = q_{r_0}$: $q = q_{R_0} + q_C$. In calculating θ_y from θ_* and $\overline{\theta}$, the velocity field of the medium in the output cross section of the furnace is taken into account. The fuel flow rate is calculated from the balance equation [2]

$$q = \mathrm{Bo}(1-\theta_y) - \mathrm{Bo}_d(\theta_y - \theta_d),$$

where

$$\mathrm{Bo} = B v \bar{c}/(F_0 \sigma T_a^3);\ \mathrm{Bo}_d = V_d \bar{c}/(F_0 \sigma T_a^3);$$

B is the fuel flow rate, kg/sec; V_d is the flow rate of the medium being drawn off at temperature θ_d.

9. Taking Account of the Spectrum

In the equations obtained for the grey bodies, the optical constants are chosen in accordance with the real spectrum. This is the usual but inadequate method of modern calculations. In [2], as an alternative, the antigrey spectrum for the CL was taken, with little influence on the results. One trend in the development of the present model is to assume the antigrey spectrum for both volume zones. The temperature field and heat fluxes may vary significantly. The heat-transfer surface is assumed to be grey.

10. Conclusion

The model proposed here for heat transfer in a furnace is distinguished by the fact that the near-wall conservative layer (CL) of the medium with a natural temperature field is taken into account. The burner is in the central volume zone and, according to the given modification of the method, may have any transmission. The radiant fluxes are described by a system of zonal equations of new type. They cannot be related to the CL temperature, since this volume zone is nonisothermal. Therefore, the balance equations are inadequate, and are completed by other equations, including the equation of internal heat transfer in the CL. The materials of the present work and [2], together with experimental data, may form the basis for practical furnace calculations.

NOTATION

a, A, r, R, D, absorptive, reflective, and transmissive capacities of volumes and surfaces; F, surface, m^2; $\bar{c}$, specific heat of the medium, $J/m^3 \cdot K$; T, temperature, K; q^*, heat-flux density, W/m^2; ρ, radius, m; v, specific flow rate of combustion products, m^3/kg; B, fuel consumption, kg/sec. Indices: 0, 1, 2, *, body surfaces, reading toward the channel axis; a, R, C, adiabatic, radiant, convective; c, e, I, r, intrinsic, effective, incident, and resultant; $q \equiv q^*/\sigma T_a^4$; $\theta = T/T_a$; $\varphi = F_*/F_0$; $F_0 = F_1$; $F_2 = F_*$.

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